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Software Implementation of an Algorithm for Solving a Dynamic Problem of Optimal Set Partitioning Under Uncertainty

Purpose. Among various formulations of the optimal set partitioning (OSP) problem, dynamic variants – where optimization conditions change over time—are of particular interest due to their relevance for real-world applications. Such systems often operate under uncertainty, which may arise from imprecise or incomplete input data, vague parameters, or unreliable mathematical representations of system behavior. This study develops a comprehensive mathematical and computational framework for solving dynamic OSP problems under uncertainty. The aim of the study is to develop software for solving a novel dynamic optimal set partitioning problem under uncertainty, specifically including the formulation of a numerical experiment, the applied interpretation of the obtained results, and a comparative analysis of the numerical experiment outcomes with the analytical results of the model investigation. **Methodology.** The methodological basis of the study consists of the principles of optimal set partitioning theory and fuzzy set theory. Modern numerical methods were used to solve systems of ordinary differential equations necessary for determining the parameters of the dynamic model. **Findings.** The formulation of dynamic optimal set partitioning problems under uncertainty allows including fuzzy model parameters and obtaining results even with incomplete information about the system. The work presents a clearly defined algorithm for solving the problem, determined by its mathematical formulation. **Originality.** The proposed models represent a significant contribution to the development of mathematical modeling, particularly in dynamic and fuzzy problem formulations. Methods and algorithms for solving the formalized problems are presented, and the results of comparative analysis allow assessing the analytical and numerical advantages of both models and the dynamic approach to solving such problems. **Practical value.** The practical value of the results obtained in this study lies in the formulation of a novel dynamic optimal set partitioning problem under uncertainty, the development of software for the numerical implementation of the experiment, and the visualization of the obtained results. The formalized mathematical model and the developed software can be applied to a wide range of practical problems, such as logistics, facility location, partitioning of communities into administrative service centers, and others.

Keywords: dynamic problem; optimal set partitioning theory; fuzzy parameter; neuro-fuzzy technologies; infinite-dimensional mathematical programming; nondifferentiable optimization; artificial intelligence methods

Introduction

Optimal set partitioning (OSP) is a fundamental problem in mathematical modeling, combinatorial optimization, and systems analysis. It involves dividing a set of elements into disjoint subsets such that a given objective function is optimized under specified constraints. Although the classical version of the problem has been extensively studied, practical applications often require consideration of more complex dynamic scenarios involving time-varying parameters, incomplete information, and structural uncertainty.

Dynamic set partitioning problems arise in many fields, including adaptive control, logistics, network planning, and decision support under uncertainty. In such systems, optimization criteria or constraints may change over time, necessitating adaptive partition structures. Input data is often fuzzy, imprecise, or insufficiently defined, rendering traditional deterministic models inadequate.

To address these challenges, hybrid approaches have been proposed that combine optimization theory with artificial intelligence methods such as fuzzy logic, neuro-fuzzy systems, and machine learning. These techniques allow for the representation and processing of uncertain information and

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enable dynamic adaptation of the partitioning process.

The aim of this study is to overcome gaps in existing literature: the lack of unified models accounting simultaneously for dynamic processes, uncertainty, and optimality criteria, and the limited application of neuro-fuzzy technologies for real-time partitioning tasks. We propose a new mathematical and computational framework consisting of two main phases: neuro-fuzzy identification to model and reduce uncertainty in input data, followed by mathematical optimization to determine the optimal partition structure under refined conditions.

The method is universal and applicable to clustering, resource allocation, and adaptive decision-making. It extends classical OSP approaches by incorporating quantitative uncertainty assessment and adaptive feedback, opening new perspectives in applied mathematics, artificial intelligence, and mathematical statistics.

Purpose

The purpose of this work is the software implementation for solving a new dynamic optimal set partitioning problem under uncertainty. Within this overall objective, the following tasks were set:

1. formalization of the new dynamic optimal set partitioning problem under uncertainty,
2. development of a solution algorithm,
3. software implementation of the algorithm,
4. analysis of the numerical and analytical results of the study.

Methodology

Dynamic and adaptive partitioning approaches have been studied in works by Lu [11], Kun Yu and Xiang Yu [10, 15], Michael A. Bender and Martín Farach-Colton [5], emphasizing the importance of flexible algorithms in real-world dynamic systems. Robust optimization under uncertainty is addressed by Barzegary and Yoganarasimhan [3], Cheramin, Hocking and Srivastava [8, 13], Moresco [4]. Hybrid frameworks and scenario-based methods were proposed by Zvara and Punyamoorty [16, 12], Zhang [14], Duong [13]. These works demonstrate that integrating dynamic, adaptive, and probabilistic components leads to more resilient and scalable models.

The theoretical foundations of optimal set partitioning were established in works by Kiselyova O. M. and her school [6, 14], showing that classical Voronoi diagrams are special cases of OSP problems. They developed algorithms for constructing additive and multiplicative Voronoi diagrams under fuzzy uncertainty using neuro-fuzzy technologies [14]. Bulat A. F. proposed a fuzzy mathematical model of set partitioning with center allocation and fuzzy element distribution [6]. These studies provide fundamental theoretical and computational tools for modeling uncertainty in partition structure and serve as a basis for the methodological innovations presented here.

Despite existing achievements, a unified model combining continuous fuzzy partition structure with adaptive real-time optimization and uncertainty handling has not yet been developed. This study aims to fill that gap by proposing a model integrating continuous and fuzzy representations, dynamic adaptation strategies, and optimization under data-driven uncertainty.

The research object is a continuous single-product dynamic optimal set partitioning problem with En without constraints, fixed subset center positions, and fuzzy parameters.

The goal is to develop a new model for this problem under uncertainty. Tasks include formulating the problem, developing solution methods and algorithms, and conducting comparative analysis of numerical experiment results.

The work uses optimal set partitioning theory for problem formalization, numerical methods for solving systems of ordinary differential equations (a key component of the model), fuzzy set theory and neuro-fuzzy technologies for posing and solving the fuzzy problem, and computer simulation for obtaining numerical results.

Findings

Problem statement. Let us consider a continuous single-product dynamic problem of optimal partitioning of a set from E_n without constraints with given locations of subset centers [14] in the following form.

Let Ω be a bounded closed Lebesgue measurable set in *n*-dimensional Euclidean space E_n.

The collection of Lebesgue measurable subsets $\Omega_1,...,\Omega_N$ of the set $\Omega \subset E_n$ (which may include

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empty ones) will be called a possible partition of this set if $\bigcup_{i=1}^{N}\Omega_i=\Omega,\ \text{mes}(\Omega_i\bigcap\Omega_j)=0$, $i,j=1,...,N$ $(i\neq j)$, where $N>0$ is a given natural number; $\text{mes}(\cdot)$ is Lebesgue measure.

Let us denote by $\sum_{\Omega}^{N}$ the class of all possible partitions of the set $\Omega\subset E_n$ into a given number N of its Lebesgue measurable subsets:

$$\sum\nolimits_{\Omega}^{N}=\Big\{\varpi\equiv\{\Omega_1,...,\Omega_N\}\in\Omega^N:$$

$$\bigcup_{i=1}^{N}\Omega_i=\Omega,\ mes\left(\Omega_i\cap\Omega_j\right)=0,\ i,j=1,...,N\left(i\neq j\right)\Big\}.$$

Next, let us denote by $\tau_1,...,\tau_N$ the collection of certain reference points for the subsets $\Omega_1,...,\Omega_N$ respectively, which we will call the centers of these subsets: $\tau_i=(\tau_i^{(1)},...,\tau_i^{(n)})\in\Omega_i$, $i=1,...,N$, and we will assume that the coordinates of all centers are given.

This work addresses two dynamic optimal set partitioning problems: one with fixed centers of subsets and the other involving the determination of the coordinates of such centers. The formalized statements of these two problems will be presented below.

Formulation of the dynamic problem with fixed centers. It is required to find a partition $\varpi=\{\Omega_1,...,\Omega_N\}\in\sum_{\Omega}^{N}$ of the set $\Omega\subset E_n$ and a vector function $c(x,\tau,t)=(c_1(x,\tau_1,t),...,c_N(x,\tau_N,t))$, defined a.e. for $x\in\Omega$ for a given fixed set of centers $\tau=\{\tau_1,...,\tau_N\}\subset\Omega^N$ and all $t\in[0,\ T]$, which ensure:

$$\inf_{\varpi\in\sum_{\Omega}^{N};\ c(\cdot,\cdot,\cdot)\in L_2^N(\Omega\times\Omega\times[0,T])} F(\varpi,\ c(\cdot,\cdot,\cdot)), \quad (1)$$

where:

$$F(\varpi,c(\cdot,\cdot,\cdot))=\int_0^T\sum_{i=1}^{N}\int_{\Omega_i}(c_i(x,\tau_i,t)\cdot m(x,\tau_i)+a_i)\cdot$$

$$\cdot\rho(x)dxdt, \quad (2)$$

subject to the conditions:

$$\frac{\partial c_i(x,\tau_i,t)}{\partial t}=\sum_{j=1}^{N}A_{ij}\cdot f_j(c_j(x,\tau_j,t)),\quad 0\le t\le T;$$

$$c_i(x,\tau_i,t_0)=c_{0i}(x,\tau_i)\quad i=1,...,N, \quad (3)$$

a.e. for $x\in\Omega$, with fixed $\tau_i=(\tau_i^{(1)},...,\tau_i^{(n)})\in\Omega_i$, $i=1,...,N$, and the closure conditions of the system:

$$\sum_{i=1}^{N}A_{ij}=1,\quad j=1,...,N. \quad (4)$$

Here $c_i(x,\tau_i,t)$, $i=1,...,N$, are the desired real-valued functions defined on $\Omega\times\Omega\times[0,T]$, which for any fixed $\tau_i=(\tau_i^{(1)},...,\tau_i^{(n)})\in\Omega_i$ are continuously differentiable with respect to the argument t on the interval $[0,T]$ a.e. for $x=(x^{(1)},...,x^{(n)})\in\Omega$, are bounded and measurable with respect to the argument x on Ω for all $t\in[0,\ T]$. $m(x,\tau_i)$, $c_{0i}(x,\tau_i)$ are given real-valued functions defined on $\Omega\times\Omega$, bounded and measurable with respect to the argument $x\in\Omega$ for any fixed $\tau_i\in\Omega_i$ for all $i=1,...,N$ (in particular, $m(x,\tau_i)$ may play the role of a metric on $\Omega\times\Omega$). $f_i(c_i(x,\tau_i,t))$, $i=1,...,N$, are given real-valued Lipschitz functions on their domain; $\rho(x)$ is a given non-negative function, bounded and measurable on Ω. a_i, $i=1,...,N$, are given, usually non-negative numbers; $0\le A_{ij}\le 1$, $i,j=1,2,...,N$, are given numerical parameters; $T>0$ and $t_0\in[0,\ T]$ are given.

Here and henceforth, the integrals are understood in the Lebesgue sense. We will assume that the measure of the set of boundary points of the subsets $\Omega_1,...,\Omega_N$ is equal to zero.

A pair $(\varpi^*,\ c^*(x,\tau,t))$, that delivers the minimal value of functional (2) on the set $\sum_{\Omega}^{N}\times L_2^N(\Omega\times\Omega\times[0,T])$ subject to constraints (3), (4), we shall consider as an optimal solution to problem (1)–(4). In this case, the partition $\varpi^*=\{\Omega_1^*,...,\Omega_N^*\}\in\sum_{\Omega}^{N}$ we shall consider as an optimal partition of the set $\Omega\subset E_n$ into N subsets, and the vector function

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$c^*(x,\tau,t)=(c_1^*(x,\tau_1,t),...,$

$c_N^*(x,\tau_N,t))\in L_2^N(\Omega\times\Omega\times[0,T])$ – as an optimal phase trajectory of the dynamical system in problem (1) – (4).

From a subject-matter point of view, the independent variable $t\in[0,\ T]$ in the given mathematical formulation of the dynamic optimal partitioning problem can play the role of the time variable, and $T>0$ and $t_0\in[0,\ T]$ are the given final and initial moments of time in the studied dynamic process, respectively. Thus, the functions $f_i(c_i(x,\tau_i,t))$, i = 1,2, …, N, in the differential relations (3), which reflect the dynamics of transportation prices, may have different forms, but the inflation/deflation model (5) will be used as the base model in this work.

$$f_i(c_i(x,\tau_i,t))=d_i\cdot c_i(x,\tau_i,t);$$
$$x\in\Omega,\tau_i=(\tau_i^{(1)},...,\tau_i^{(n)})\in\Omega_i; \qquad (5)$$
$$i=\overline{1,N},0\le t\le T.$$

In formula (5) d_i, i=1, ..., N, are given real coefficients reflecting the tendency of the transportation cost of a unit of product from the i-th center to the consumer to increase/decrease/.

Problem Statement under Uncertainty. In problem (1)–(2) under conditions (3)–(4) (the crisp problem), it was assumed that all parameters are given as constants. However, in practice, the exact values of these constants are usually unknown or specified linguistically (fuzzily). Let us consider the parameters in more detail $a_1,...,a_N$ in the objective functional (2), these parameters represent fixed costs for production and transportation tasks. Depending on the circumstances, fixed costs may include factory operation expenses, insurance, and possibly maintaining a minimum number of employees. These costs are constant and remain unchanged regardless of the firm's production volume. However, in many practical production and transportation problems, the values of these parameters (fixed costs) depend on numerous real-world factors not accounted for in the given model. A model with fixed parameter values $a_1,...,a_N$ may prove to be too «coarse», since in practice, the known information often consists not of precise exact parameter values, but sets of their possible values.

To address uncertainty in specifying parameters $a_1,...,a_N$ in the objective functional (2), we will consider them as fuzzy variables, which, in turn, depend on fuzzy factors $\gamma=(\gamma_1,...,\gamma_n)$. Such factors for fixed production costs may include the costs of production means, labor, natural resources, and others. Accounting for this additional information complicates the original mathematical model but, nevertheless, it may prove to be acceptably more accurate and adequate because it considers the influence of additional factors on the parameters $a_1,...,a_N$.

To formalize the fuzziness, we will subsequently use neuro-fuzzy technologies. For this purpose, the parameters a_i, $i=1,...,N$, will be represented as parameters dependent on fuzzy factors γ_j, $j=1,...,n,$ in the form of

$$a_i\equiv a_i(\gamma_1,...,\gamma_n). \qquad (6)$$

Then the objective functional (2) with fuzzy parameters (6) can be expressed in the following form:

$$J(\lambda(\cdot),c(\cdot,\cdot,\cdot))=\int_0^T\int_\Omega\sum_{i=1}^N(c_i(x,\tau_i,t)\cdot m(x,\tau_i)+$$
$$+a_i(\gamma_1,...,\gamma_n))\rho(x)\lambda_i(x)dxdt; \qquad (7)$$

$$a(\gamma)=\frac{\sum_{k=1}^L d_k\cdot\mu_{D_k}^*(a)}{\sum_{k=1}^L\mu_{D_k}^*(a)},\gamma\in\Upsilon_1\times...\times\Upsilon_i\times...\times\Upsilon_n, \qquad (8)$$

where:

$$\mu_{D_k}^*(a)=\begin{cases}\sum_{j=1}^{S_k}p_j^{k*}(\gamma_1,\gamma_2,...,\gamma_n),\\ if\sum_{j=1}^{S_k}p_j^{k*}(\gamma_1,\gamma_2,...,\gamma_n)\le 1,\\ 1, otherwise;\end{cases} \qquad (9)$$

$$p_j^{k*}(\gamma_1,\gamma_2,...,\gamma_n)=w_j^{k*}\prod_{i=1}^n\mu_{ij}^{k*}(\gamma_i); \qquad (10)$$

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$$\mu_{ij}^{k*}(\gamma_i) = \frac{1}{1+\left(\frac{\gamma_i - b_{ij}^{k*}}{c_{ij}^{k*}}\right)^2}, \quad (11)$$

$i = 1,...,n,\ j = 1,...,s_k, k = 1,...,L$

Solution Algorithm. 1. Set Ω we include in n-dimensional parallelepiped Π, whose sides are parallel to the axes of the Cartesian coordinate system. The parallelepiped Π is covered by a rectangular grid.

2. We set the values of the functions $c_{0i}(x,\tau_i),\ i = 1,...,N$, from the initial conditions (3) at the grid nodes.

3. We cover the interval $[0,T]$ with a grid of step size h_t.

4. For each grid node and for each $\tau_i,\ i = 1,...,N$ we solve the Cauchy problem for the system of ordinary differential equations (3) over the time interval $[0,T]$ and find the functions $c_i(x,\tau_i,t),\ i = 1,...,N$.

5. We calculate the values of the parameters $a_i,\ i = 1,...,N$, using the formulas (8) – (11).

6. We define the characteristic function $\lambda^*(x)$ at the grid nodes from step 1 with the obtained $c_i(x,\tau_i,t),\ i = 1,...,N$ and the fuzzy parameter values restored in step 5 $a_i,\ i = 1,...,N$.

7. We find the minimum value of the objective functional (2) using the obtained $\lambda^*(x)$ and $c_i^*(x,\tau_i,t),\ i = 1,...,N$.

Results of the numerical experiment. This paragraph presents several pairs from numerous computational experiments conducted for models (1)–(4) and (9)–(11).

To construct the optimal partition, the domain Ω was covered with a rectangular grid with nodes (i, j), $i, j = 1,...101$. For the obtained points of the phase trajectory, the transportation cost from each node was calculated (i, j) set Ω for centers τ_1, τ_2. Based on the minimum cost criterion, the node (i, j) was assigned to the sets Ω_1 and Ω_2 respectively. The accuracy of the r-algorithm, both for the variables and the subgradient, was the same and amounted to 10^{-4} Figures 1–3 show the optimal partitioning of the set of consumers Ω_1, which has the shape of a square with side length 1, into two subsets with centers $\tau_1(0.5;0.25)$, $\tau_2(0.5;0.75)$. The set at the bottom of the figures – Ω_1; The set at the top of the figures – Ω_2.

Figures 1–3, *a* show the optimal partitioning for exact values at $a_1 = 3$; $a_2 = 1$. Figures 1–3, *b* for the restored values before tuning the fuzzy model at $a_1 = 2.6246$; $a_2 = 1.9613$. Figures 1–3, *c* for the restored values after tuning the fuzzy model at $a_1 = 3.0604$; $a_2 = 0.9828$

a *b* *c*

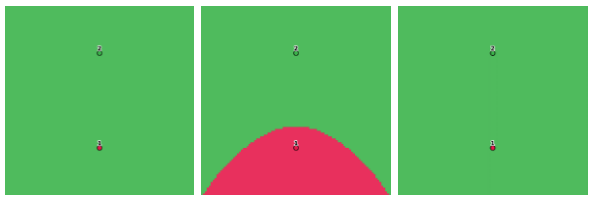

Fig. 1. Optimal partitioning Ω over the time interval [0; 1] at c_1=4.6967; c_2=5.0451

a *b* *c*

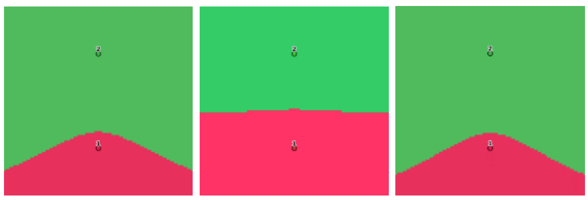

Fig. 2. Optimal partitioning Ω over the time interval [0; 2] at c_1=11.9758; c_2=15.9638

a *b* *c*

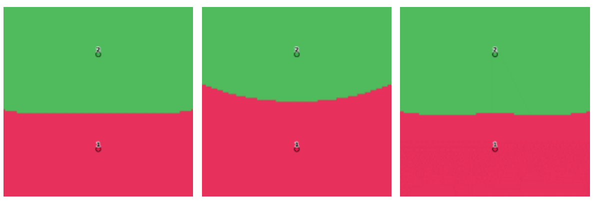

Fig. 3. Optimal partitioning Ω over the time interval [0; 3] at c_1=29.7873; c_2=42.6809

The numerical results of the algorithm's performance are presented in Table 1 at initial values of variable costs $c_{01} = 2; c_{02} = 1$.

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Table 1

Numerical results of the algorithm's

Time interval, variable costs	For exact values $a_1 = 3$; $a_2 = 1$	For the restored values before the adjustment $a_1 = 2.625$; $a_2 = 1.961$	For the restored values after the adjustment $a_1 = 3.06$; $a_2 = 0.982$
[0; 1] $c_1 = 4.697$ $c_2 = 5.045$	$F = 2.153$ $S_1 = 0$ $S_2 = 1$	$F = 3.043$ $S_1 = 0.236$ $S_2 = 0.764$	$F = 2.136$ $S_1 = 0$ $S_2 = 1$
[0; 2] $c_1 = 11.976$ $c_2 = 15.964$	$F = 6.909$ $S_1 = 0.246$ $S_2 = 0.754$	$F = 7.892$ $S_1 = 0.446$ $S_2 = 0.554$	$F = 6.912$ $S_1 = 0.231$ $S_2 = 0.769$
[0; 3] $c_1 = 29.787$ $c_2 = 42.681$	$F = 15.699$ $S_1 = 0.441$ $S_2 = 0.558$	$F = 16.639$ $S_1 = 0.53$ $S_2 = 0.47$	$F = 15.749$ $S_1 = 0.434$ $S_2 = 0.566$

Originality and practical value

The scientific novelty of the presented works lies, in particular, in the formulation and formalization of a new dynamic optimal set partitioning problem under uncertainty, the development of a solution algorithm, the design of a numerical experiment, and the analysis of the obtained numerical and analytical results. The results obtained in this study can be applied to solve a wide range of practical placement and partitioning problems in a more refined and adequate formulation for applied needs, particularly in logistics, transportation, and the partitioning of large community territories into administrative service centers.

Conclusions

The study is devoted to the software implementation of the dynamic problem of optimal set partitioning with fixed centers under uncertainty. A detailed review of existing approaches is provided, highlighting the specific features of dynamic formulations of optimal set partitioning problems, particularly under uncertainty, and emphasizing their key advantages over classical formulations. A numerical experiment was conducted, allowing visual assessment and comparison between crisp and fuzzy partitioning before and after the tuning of fuzzy parameters. A comparative analysis was performed to provide practical justification.

The graphs presented in the study were obtained using software developed by the authors in Python, based on various values of the model’s input parameters, some of which were derived from solving system (3). The results confirmed the adequacy of the proposed model, and the outcomes of the numerical experiment allow for a visual evaluation of the model's validity. Based on these results, it can be concluded that the dynamic and fuzzy formulation of the optimal partitioning problem significantly enhances the adequacy of the mathematical model and makes it more adaptable for practical applications.

Time-varying system parameters have a significantly negative impact on the accuracy of robust models, but their inclusion in the problem formulation helps mitigate this influence. Further research may focus on more complex formulations of dynamic optimal partitioning problems under uncertainty, particularly in cases involving the placement of subset centers and constraints on network capacity.

LIST OF REFERENCE LINKS

1. Яковлев С. В., Кісельова О. М,, Гарт Л. Л., Кузенков О. О., Закутній Д. В. Про динамічну задачу оптимального розбиття множин з фіксованими центрами. *Сучасні проблеми моделювання*. 2025. № 28. С. 190–203. DOI: https://doi.org/10.33842/2313-125x-2025-30-190-203
2. Adufu T., Kim Y. Dyna-P: Placement-aware Dynamic Partitioning for Lightweight Applications with Modern GPUs. *Research Square*. 2025. DOI: https://doi.org/10.21203/rs.3.rs-5774593/v1
3. Barzegary E., Yoganarasimhan H. A Recursive Partitioning Approach for Dynamic Discrete Choice Modeling in High-Dimensional Settings. *arXiv*. 2025. URL: https://arxiv.org/pdf/2208.01476
4. Bender M. A., Farach-Colton M., Goodrich M. T., Komlós H. History-Independent Dynamic Partitioning: Operation-Order Privacy. *ACM Transactions on Privacy and Security*. 2024. Vol. 2, Iss. 2. P. 1–27. DOI: https://doi.org/10.1145/3651609

doi: https://doi.org/10.15802/stp2025/342252

5. Bulat A., Kiseleva E., Yakovlev S., Prytomanova O., Lebediev D. Solving the Problem of Fuzzy Partition-Distribution with Determination of the Location of Subset Centers. *Computation*. 2024. Vol. 12, Iss. 10. P. 199. DOI: https://doi.org/10.3390/computation12100199
6. Cheramin M., Chen R. L.-Y., Cheng J., Pinar A. Data-driven robust optimization using scenario-induced uncertainty sets. *arXiv*. 2021. DOI: https://doi.org/10.48550/arXiv.2107.04977
7. Hocking T. D., Srivastava A. Labeled Optimal Partitioning. *arXiv*. 2020. DOI: https://doi.org/10.48550/arXiv.2006.13967
8. Kiseleva E. M., Prytomanova O. M., Hart L. Solving a Two-stage Continuous-discrete Problem of Optimal Partitioning-Allocation with Subsets Centers Placement. *Open Computer Science*. 2020. Vol. 10, Iss. 1. P. 124–136. DOI: https://doi.org/10.1515/comp-2020-0142
9. Kuzenkov O., Kostenko O. Software development for solving the dynamic problem of optimal set partitioning with determination of the coordinates of their centers. *Актуальні проблеми автоматизації та інформаційних технологій*. 2024. № 28. С. 306–317. DOI: https://doi.org/10.15421/432429
10. Kuzenkov O., Lozovskyi A. Software development for solving the dynamic problem of optimal set partitioning with fixed centers. *Актуальні проблеми автоматизації та інформаційних технологій*. 2023. № 27. С. 212–220. DOI: https://doi.org/10.15421/432320
11. Lu J., Zhou T., Osorio C. Adaptive partitioning strategy for high-dimensional discrete simulation-based optimization problems. *arXiv*. 2021. DOI: https://doi.org/10.48550/arXiv.2104.14119
12. Moresco M., Mailhot M., Pesenti S. M. Uncertainty propagation and dynamic robust risk measures. *Mathematics of Operations Research*. 2024. Vol. 50, Iss. 3. P. 1939–1964. DOI: https://doi.org/10.1287/moor.2023.0267
13. Sudoso A., Aloise D. A Column Generation Algorithm with Dynamic Constraint Aggregation for Minimum Sum-of-Squares Clustering. *INFORMS Journal on Computing*. 2025. DOI: https://doi.org/10.1287/ijoc.2024.0938
14. Yu K., Zhang W. Dynamic data partitioning strategy for distributed learning on heterogeneous edge system. *Computer Communications*. 2025. Vol. 241. 108262. DOI: https://doi.org/10.1016/j.comcom.2025.108262
15. Yu X., Zhai H., Tian R., Guan Y., Polat K., Alhudhaif A. Dynamic trajectory partition optimization method based on historical trajectory data. *Applied Soft Computing*. 2024. Vol. 151. 111120. DOI: https://doi.org/10.1016/j.asoc.2023.111120
16. Zvara Z., Szabó P. G. N., Lóránt B. B., Benczúr A. A. System-aware dynamic partitioning for batch and streaming workloads. *Proceedings of the 14th IEEE/ACM International Conference on Utility and Cloud Computing*. 2021. P. 1–10. DOI: https://doi.org/10.1145/3468737.3494087

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Програмна реалізація алгоритму розв'язання динамічної задачі оптимального розбиття множин в умовах невизначеності

Мета. Серед різних формулювань задачі оптимального розбиття множини (OSP) особливий інтерес представляють динамічні варіанти, в яких умови оптимізації змінюються з часом, через їхню актуальність для реальних застосувань. Такі системи часто працюють в умовах невизначеності, яка може виникати через неточні або неповні вхідні дані, нечіткі параметри або ненадійні математичні представлення поведінки системи. У цьому дослідженні розробляється комплексна математична та обчислювальна основа для вирішення динамічних задач OSP в умовах невизначеності. Метою дослідження є розробка програмного забезпечення

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для вирішення нової динамічної задачі оптимального розбиття множин в умовах невизначеності, зокрема, включаючи формулювання чисельного експерименту, прикладну інтерпретацію отриманих результатів та порівняльний аналіз результатів чисельного експерименту з аналітичними результатами дослідження моделі. **Методика.** Методологічна основа дослідження складається з принципів теорії оптимального розбиття множин та теорії нечітких множин. Для вирішення систем звичайних диференціальних рівнянь, необхідних для визначення параметрів динамічної моделі, були використані сучасні чисельні методи. **Результати.** Формулювання динамічних задач оптимального розбиття множин в умовах невизначеності дозволяє включати нечіткі параметри моделі та отримувати результати навіть за наявності неповної інформації про систему. У роботі представлено чітко визначений алгоритм вирішення задачі, що визначається її математичним формулюванням. **Наукова новизна.** Запропоновані моделі є значним внеском у розвиток математичного моделювання, зокрема в динамічних та нечітких формулюваннях задач. Представлено методи та алгоритми розв'язання формалізованих задач, а результати порівняльного аналізу дозволяють оцінити аналітичні та чисельні переваги обох моделей та динамічного підходу до розв'язання таких задач. **Практична значимість.** Практична значимість отриманих у цій роботі результатів полягає у формулюванні нової динамічної задачі оптимального розбиття множини в умовах невизначеності, розробці програмного забезпечення для чисельної реалізації експерименту та візуалізації отриманих результатів. Формалізована математична модель та розроблене програмне забезпечення можуть бути застосовані до широкого кола практичних задач, таких як логістика, розміщення об'єктів, розбиття громад на адміністративні центри обслуговування та інші.

Ключові слова: динамічна задача; теорія оптимального розбиття множин; нечіткий параметр; нейронечіткі технології; нескінченновимірне математичне програмування; недиференційована оптимізація; методи штучного інтелекту

REFERENCES

1. Yakovlev, S., Kiseleva, E., Hart, L., Kuzenkov, O., & Zakutnii, D. (2025). On the dynamic problem of optimal set partitioning with fixed centers. *Modern Problems of Modeling*, *28*, 190-203. DOI: https://doi.org/10.33842/2313-125x-2025-30-190-203 (in Ukrainian)
2. Adufu, T., & Kim, Y. (2025). Dyna-P: Placement-aware Dynamic Partitioning for Lightweight Applications with Modern GPUs. *Research Square*. DOI: https://doi.org/10.21203/rs.3.rs-5774593/v1 (in English)
3. Barzegary, E., & Yoganarasimhan, H. (2025). A Recursive Partitioning Approach for Dynamic Discrete Choice Modeling in High-Dimensional Settings. *arXiv*. URL: https://arxiv.org/pdf/2208.01476 (in English)
4. Bender, M. A., Farach-Colton, M., Goodrich, M. T., & Komlós, H. (2024). History-Independent Dynamic Partitioning: Operation-Order Privacy. *ACM Transactions on Privacy and Security*, *2*(2), 1-27. DOI: https://doi.org/10.1145/3651609 (in English)
5. Bulat, A., Kiseleva, E., Yakovlev, S., Prytomanova, O., & Lebediev, D. (2024). Solving the Problem of Fuzzy Partition-Distribution with Determination of the Location of Subset Centers. *Computation*, *12*(10), 199. DOI: https://doi.org/10.3390/computation12100199 (in English)
6. Cheramin, M., Chen, R. L.-Y., Cheng, J., & Pinar, A. (2021). Data-driven robust optimization using scenario-induced uncertainty sets. *CoRR*. DOI: https://doi.org/10.48550/arXiv.2107.04977 (in English)
7. Hocking, T. D., & Srivastava, A. (2020). Labeled Optimal Partitioning. *arXiv*. DOI: https://doi.org/10.48550/arXiv.2006.13967 (in English)
8. Kiseleva, E. M., Prytomanova, O. M., & Hart, L. (2020). Solving a Two-stage Continuous-discrete Problem of Optimal Partitioning-Allocation with Subsets Centers Placement. *Open Computer Science*, *10*(1), 124-136. DOI: https://doi.org/10.1515/comp-2020-0142 (in English)
9. Kuzenkov, O., & Kostenko, O. (2024). Software development for solving the dynamic problem of optimal set partitioning with determination of the coordinates of their centers. *Actual Problems of Automation and Information Technology*, *28*, 306-317. DOI: https://doi.org/10.15421/432429 (in English)
10. Kuzenkov, O., & Lozovskyi, A. (2023). Software development for solving the dynamic problem of optimal set partitioning with fixed centers. *Actual Problems of Automation and Information Technology*, *27*, 212-220. DOI: https://doi.org/10.15421/432320 (in English)
11. Lu, J., Zhou, T., & Osorio, C. (2021). Adaptive partitioning strategy for high-dimensional discrete simulation-based optimization problems. *arXiv*. DOI: https://doi.org/10.48550/arXiv.2104.14119 (in English)
12. Moresco, M., Mailhot, M., & Pesenti, S. M. (2024). Uncertainty propagation and dynamic robust risk measures. *Mathematics of Operations Research*, *50*(3), 1939-1964. DOI: https://doi.org/10.1287/moor.2023.0267 (in English)

doi: https://doi.org/10.15802/stp2025/342252

13. Sudoso, A., & Aloise, D. (2025). A Column Generation Algorithm with Dynamic Constraint Aggregation for Minimum Sum-of-Squares Clustering. *INFORMS Journal on Computing*. DOI: https://doi.org/10.1287/ijoc.2024.0938 (in English)
14. Yu, K., & Zhang, W. (2025). Dynamic data partitioning strategy for distributed learning on heterogeneous edge system. *Computer Communications*, *241*, 108262. DOI: https://doi.org/10.1016/j.comcom.2025.108262 (in English)
15. Yu, X., Zhai, H., Tian, R., Guan, Y., Polat, K., & Alhudhaif, A. (2024). Dynamic trajectory partition optimization method based on historical trajectory data. *Applied Soft Computing*, *151*, 111120. DOI: https://doi.org/10.1016/j.asoc.2023.111120 (in English)
16. Zvara, Z., Szabó, P. G. N., Lóránt, B. B., & Benczúr, A. A. (2021). System-aware dynamic partitioning for batch and streaming workloads. *Proceedings of the 14th IEEE/ACM International Conference on Utility and Cloud Computing*, 1–10. DOI: https://doi.org/10.1145/3468737.3494087 (in English)

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