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RAILWAY TRACK REPRESENTATION IN MATHEMATICAL MODEL OF VEHICLES MOVEMENT

Purpose. The tasks of modeling the interaction of track and rolling stock are basic ones for most areas of modern scientific research of railway transport. The compilation of the model by the principle of Lagrange d'Alembert has found a very wide application for solving the problems of rolling stock dynamics. Representation of the railway track in the model of vehicle movement can be implemented in several ways, which, among other things, will differ in detail. The purpose of this work is to create a methodology for representing the railway track in mathematical models of interaction with rolling stock and obtaining practical results for different characteristics and design of the track and the level of maximum speed. **Methodology.** The problem consists of determining such track characteristics as the reduced mass, the stiffness coefficient, and the dissipation coefficient. As a tool for solving this problem we used the model of the stress-strain behavior of the railway track based on the joint use of the elastic wave propagation equations to describe the topography of the part of the system that is involved in the interaction at a given time and the equations of dynamic equilibrium of its deformation. This makes it possible to take into account the dynamics of the deflection of the underrail base, which is especially important for the conditions of passenger traffic, which can be carried out at high speed. **Findings.** The authors obtained theoretically justified stiffness and dissipation coefficients of the railway track for calculating the dynamics of rolling stock in modern models based on systems of equations in accordance with the Lagrange d'Alembert principle. The established values, in contrast to those given in other sources, have a reasonable dependence on the track design and the speed of movement. **Originality.** The authors expanded the approaches of railroad track representation in models of rolling stock described by systems of equations by the Lagrange-d'Alembert principle. The paper presents the developed method for determining the characteristics of the railway track for such models is based on the results of variant calculations of the dynamic deflection of the rail from the passage of the wheel. **Practical value.** The authors obtained the values of the stiffness and dissipation coefficients of the railway track depending on the design and speed of motion for practical use in appropriate models of interaction between track and rolling stock.

Key words: railway track; interaction of track and rolling stock; railway track model; track stiffness; track dissipation; dynamic track deflection; passenger traffic

Introduction

The tasks of modeling the interaction of track and rolling stock are the basic ones for most areas of modern scientific railway transport research. Depending on the task being solved, one can use both relatively simple, sometimes even flat calculation schemes and the developed models de-

scribed by systems with dozens of equations. Despite the fact that it is always being modeled the interaction process between track and rolling stock, the tasks of rolling stock research and those aimed at the railway track research have fundamental differences.

Today one can not recommend a single mathematical model and, even, a single modeling

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principle to solve the entire variety of tasks of interaction between the track and rolling stock. It is the problem definition should determine the permissible hypotheses and assumptions, obligatory factors of influence, possible accuracy of the output data and sufficient accuracy of the obtained results and other characteristics determining the choice of the adequate for this case model.

It is rather difficult to obtain satisfactory results trying to combine simulation of rolling stock and railway track operation. Therefore, the most commonly used are not general models, but a staged modeling of processes in several different models. Thus, in the models of rolling stock with detailed accounting of dynamic processes in a vehicle and maximal simplification of rail track, the forces acting on the track are obtained. Further, such forces become the initial load (usually a simplified one to the static application of these forces) in models with detalization of rail track operation. This approach to the consistent application of two fundamentally different models is becoming increasingly widespread today.

Models of rolling stock are, in most cases, systems of motion (oscillations) of a set of interconnected solids. Usually, for the mathematical description of such models, the systems of differential equations of the second kind, compiled according to Lagrange-d'Alamber principle are used [1, 2].

As a rule, it is taken a constant mass of interacting bodies, linearity of the connections between them, constant speed of motion, immutability of geometry of the bodies themselves (the deformations of system are determined by the change in position of bodies, not by their own deformation), the invariance of the physical (especially the elastic ones) characteristics of bodies and so on. Attempts to eliminate such assumptions considerably complicate solution of the equation system, and in most cases, it makes impossible to obtain the solution with the necessary accuracy at all.

Particular attention should be paid to the fact that the system of bodies, described by the Lagrange equations of the second kind may have a local or global coordinate system. From the point of view of mathematical tools, this does not have a fundamental difference, but it has significant disadvantages for practical application. The local coordinate system assumes that its center moves together with the system, as a rule it is located in the center of the weight of the largest body, for

example, the car body. Then the coordinates of all bodies at each step of calculation are moves relative to this point. At each step of calculation, they have values about one order, both between themselves and between values at other steps. This approach provides a certain accuracy of results of solving the equation system at each step in each direction. However, with this approach, a vehicle under study does not move along the track, but only fluctuates in one place. Local track inequalities are set in the same local coordinate system, and the global change in the motion trajectory is taken into account as application of the corresponding external forces. Thus, for example, the motion in a curve is set by applying to the bodies of the corresponding centrifugal force.

Global coordinate system establishes a permanent point of its center, the position of which does not change during the entire sequence of calculations. It simplifies the mechanism for describing the track position and makes it possible to set its spatial position as an array of coordinates. The advantages of this approach are, first, the ability to set any layout of the track, including the results of outdoor shooting; and secondly, the forces arising from changing the motion trajectory will be calculated as a consequence, not set as the source data. However, all displacements of individual bodies of car will also be determined relative to the originally set coordinate center. This will increase the calculation error.

Consequently, for the tasks in which it is the rolling stock dynamics that is under study, the local coordinate system is used - this is a more common variant. In some cases, when it is important to study the motion dynamics, depending exactly on the track geometry the global coordinate system is used.

Modeling according to the principle of Lagrange-d'Alamber has found a very broad application for solving the tasks of rolling stock dynamics. In accordance with the principles of solid state mechanics it is assumed that the object has a constant mass. The force applied to the object point instantly leads to displacement of all its other points and the body moves as one whole relative to the mass center. The use of such an approach for modeling the operation of a railway track construction may take place, but for the most tasks it should be considered inappropriate. One can divide a railway track into integral objects with a constant mass on-

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ly conditionally. Moreover, these masses will have small displacement values, which will occur in a short period of time.

Purpose

The work is aimed to create a methodology for representing a rail track in mathematical models of interaction with rolling stock and obtain practical results for various characteristics and track design and the maximum speed level.

Methodology

Rail track representation in the model of vehicle movement can be carried out in several ways, which, first of all, will differ in different detalization.

One of the variants involves description of rail track as an endless beam, which lies on an elastic, non-inertial basis – the Winkler model. This model is the basis of many static calculation schemes and is adequate for many tasks. But such a system will have an infinite number of freedom degrees, which is inconvenient for inclusion in rolling stock models. An alternative is the Vlasov model, which makes it possible to express displacement of the beam (rail) points and underrail base through displacement of the contact points of wheels and rails.

In this case, the general approach is maintained: a system assembly of separate rigid bodies with mass and connections between them. In the simplest form it can be a tight connection of wheel with a rail track, Fig. 1, *a*. Then the stiffness of such a bearing reduced to one wheel can be determined by the formula

$$k_t = \frac{2U}{k}, \quad (1)$$

where U – is an elasticity modulus of underrail base; k –relative stiffness coefficient.

This variant is rarely used. The absence of a dissipative connection makes the system too sensitive to the swing: the accumulation of errors in the solution of equations can lead to continuous oscillations.

In most cases, it is used the variant where each wheel has a rigidly dissipative connection with the reduced part of the rail track – Fig. 1, *b*.

Today, it is spreading the tendency to take into account the track inertia, that is, a track (or its individual elements) should have a mass, which

makes it an integral object of the mass oscillation system and makes it possible to detalize for separate components. The weight implies the so-called «reduced mass» – that is, the one interacting with the wheel. Sometimes it means the mass of rail, sometimes – the mass of the track upper structure (again, reduced to one wheel). It is clear that in any case this mass is conditional, its value should change in the process of interaction, but systems based on the Lagrange-d'Alamber princile make it impossible to use variable masses. In addition, the railway track operates in the conditions of elastic deformations that can not be fully identified by the displacement of mass centers of rigid bodies.

Depending on the detalization degree of subsystem «rail track», the following variants are possible:

- the reduced mass of rail is taken into account; it is assumed that the rail and the wheel, as an object with their total mass, have a rigidly dissipative connection with underrail base, Fig. 1, *c*;

- it is taken into account the reduced mass of rail, which has a rigidly dissipative connection with underrail base; to separate the wheel mass from the rail between them a rigid connection, (with a large numerical value) is established, Fig. 1, *d*;

- it is taken into account the reduced mass of the track is taken into account, which has a rigidly dissipative connection with the base, Fig. 1, *e*;

- it is taken into account the weight of sleeper (interacting with the wheel), which has a rigidly dissipative connection with under sleeper base on the one side and with a rail on the other (as a rule, unimportant one (Fig. 1, *f*), although there may be the variants of rail mass separation too (Fig. 1, *g*);

- it is taken into account the weight of sleeper (interacting with the wheel), which has a rigidly dissipative connection with the rail on the one side and with ballast on the other, which has a rigid connection with sub-ballast base (Fig. 1, *f*).

The given classification is developed on the basis of works [1, 2, 6, 7, 9–11], and others.

From the point of view of vehicle movement modeling the variants from "f" (see Fig. 1) and further are not specify the calculation results as they complicate the general system of equations and can be identically referred to the variant "e" or even simpler ones. Such detalization takes place when trying to simulate the railroad operation itself.

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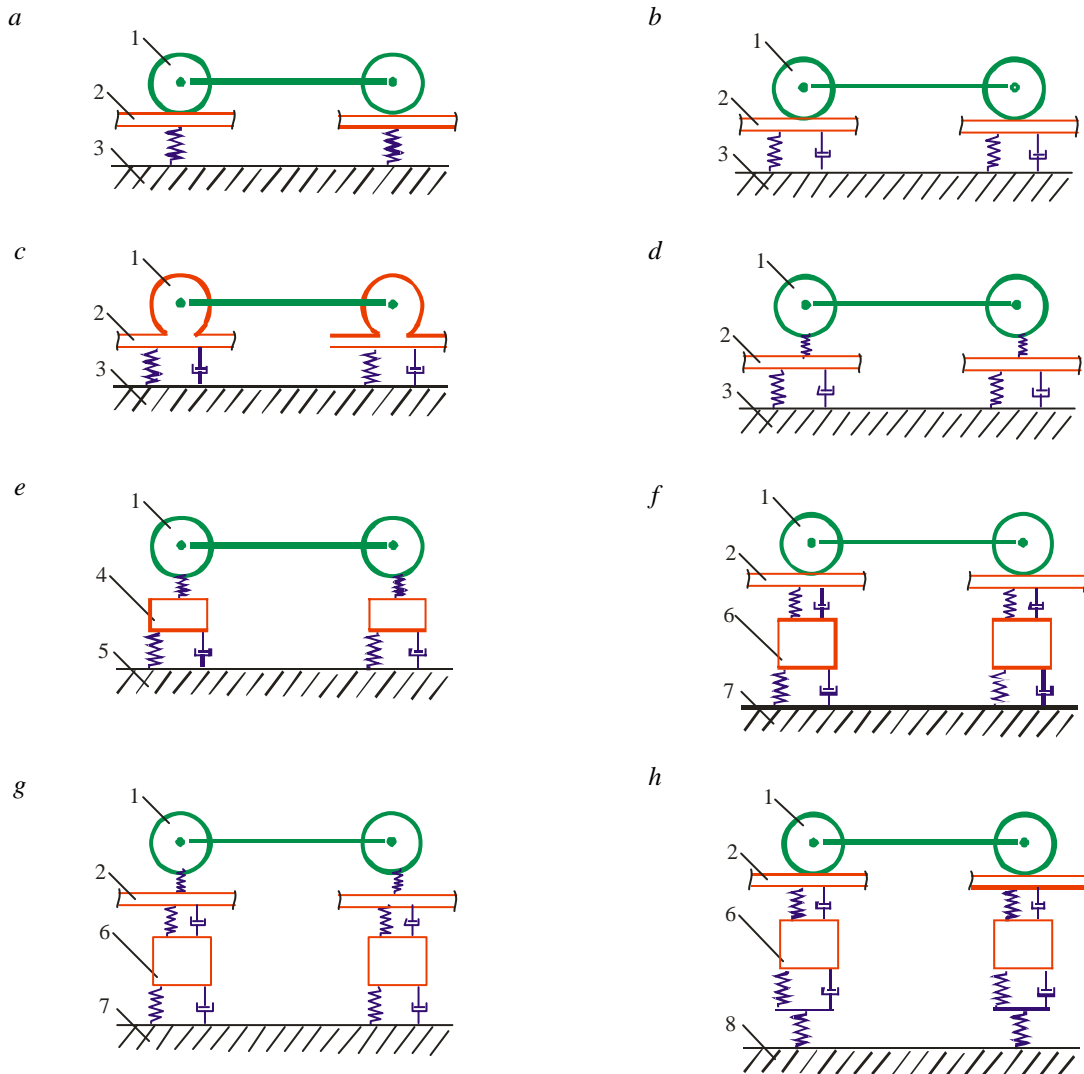


Fig. 1. Options for rail track representation in the model based on the Lagrange-d'Alembert equation system:

1 – wheel; 2 – rail; 3 – underrail base; 4 – conditional object with a weight of a track, reduced to one wheel; 5 – immovable foundation; 6 – sleeper; 7 – under sleeper base; 8 – sub-ballast base

Thus, the task of presenting a rail track in models of rolling stock is reduced to determining characteristics of connections between its elements and the wheel. They can be obtained either by purely analytical approaches, or according to the results of dependences of the track deflections on the applied load. The latter variant can be implemented either experimentally according to the results of field measurements, or theoretically based on the results of variant calculations using corresponding models of the stress-strain behavior of the rail track. A similar problem was solved by

Prof. O. M. Darenskyi in the work [3]. This work based on analytical calculations determines the support rigidity reduced to one wheel, for the zone of rail joints in the conditions of industrial transport operation.

In order to solve such problem for mainline transport, especially for the conditions of passenger traffic, which can occur at high speed [4], one of the important characteristics of the rail track model which can be applied is the ability to take into account the dynamics of the underrail base deflection.

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Thus, the task consists of determining such characteristics of the track as the reduced mass, rigidity coefficient, and dissipation coefficient. To solve the task let us consider the following calculation scheme, Fig. 2.

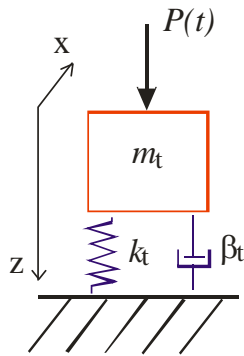


Fig. 2. Calculation scheme of track operation reduced to one wheel

Operation of the track reduced to one wheel is considered as a system with one freedom degree consisting of mass that has a rigid (k_t) and dissipative (β_t) connection with the base. An external force (P) variable in time is applied to the system. Such calculation scheme is identical to the rail track representation in the Lagrange-d'Alembert models shown in the Fig. 1, e, but, if necessary, it can be reduced to the other variants.

$$z(x) = Ce^{-rx} (\cos \omega x + \sin \omega x) + \frac{A}{m_t V^2} \frac{[(\omega^2 + 2k_t^2 - 4rk_t) \cos k_t x + (\omega^2 - 2k_t^2) \sin k_t x] e^{-k_t x}}{\omega^4 - 4rk_t \omega^2 + 4k_t^4 + 8r^2 k_t^2 - 8rk_t^3}, \quad (6)$$

$$\text{where } C = \frac{A}{m_t V^2} \left(\frac{k}{2U} - \frac{\omega^2 + 2k_t^2 - 4rk_t}{\omega^4 - 4rk_t \omega^2 + 4k_t^4 + 8r^2 k_t^2 - 8rk_t^3} \right).$$

Let's assume that the wheel covers the distance from one axle between sleepers to the other, Fig. 3, and then the process is cyclically repeated. This approach corresponds both to the models in which the vehicle (wheel) movement along the track is conditional (wheel position relative to the track does not change in the local coordinate system, and movement relative to the track is taken into account by applying the corresponding external forces (accelerations, constraints), and the models in which movement along the track is set in explicit form, taking into account the wheel position along the track length.

Differential equation for describing fluctuations of such system will have the following form

$$m_t \frac{d^2 z}{dt^2} + \beta_t \frac{dz}{dt} + k_t z = P(t). \quad (2)$$

If one integrates along the length of the covered distance, then

$$m_t V^2 \frac{d^2 z}{dx^2} + \beta_t V \frac{dz}{dx} + k_t z = P(x), \quad (3)$$

where V – is the movement speed.

The equations (3) are reduced to the classical form

$$\frac{d^2 z}{dx^2} + 2r \frac{dz}{dx} + \omega^2 z = \frac{P(x)}{H}, \quad (4)$$

$$\text{where } r = \frac{\beta_t}{2m_t V}; \omega = \sqrt{\frac{k_t}{m_t V^2}}; H = m_t V^2.$$

The force will be presented using the expression

$$P(x) = Ae^{-kx} (\cos kx + \sin kx), \quad (5)$$

where A – is the parameter determining the force amplitude.

Then solution of the equation (4) will have following form:

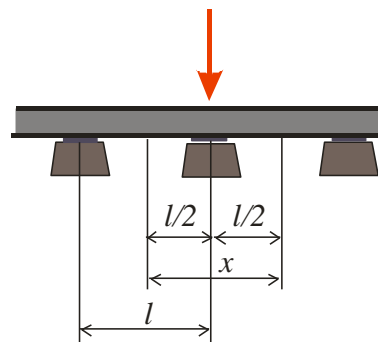


Fig. 3. Sample limits for deflection dependence on the strength

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As a tool for solving such a problem it was used a model of stress-strain behavior of the rail track based on combination of equations of elastic wave propagation to describe the geometry of the outline of the part of the system space that is involved in the interaction at a given time and the equations of dynamic equilibrium of its deformation [5].

Findings

According to simulation results of the dynamic deflection of the rail, one can establish its dependence on the applied force. As noted above, this dependence will be determined in the area $x \in [0 \pm l/2]$, where l – is the distance between the sleeper axes. Example of the obtained deflections when the support is load-unloaded by a wheel for the area with elasticity modulus of underrail base of 32 MPa is shown in the Fig. 4

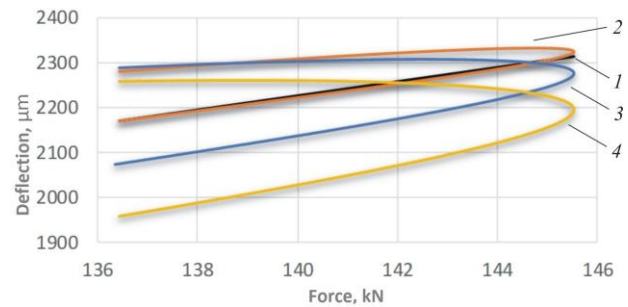


Fig. 4. Deflection of rail track according to modeling:

1 – statics; 2 – 80 km/h; 3 – 160 km/h; 4 – 240

Formula (6) shows the analytic dependence of the deflection on the force. Thus, the track characteristics can be determined as a result of approximating the data array of deflections obtained by modeling (Z_m) by function (6). Approximation algorithm according to criterion of the least squares:

$$\left. \begin{aligned} z &= f(k_t, \beta_t, m_t, P); Z_m = \{z_{m(i)} = f(P)\}; R = \sum_i (z_{m(i)} - z)^2; \\ \exists k_t &\in (k_{t(\min)}; k_{t(\max)}), \exists \beta_t \in [0; \beta_{t(\max)}], \exists m_t \in [0; m_{t(\max)}]: R \rightarrow \min; \\ k_t &\neq k_{t(\min)}; k_t \neq k_{t(\max)}; \beta_t \neq \beta_{t(\max)}; m_t \neq m_{t(\max)}. \end{aligned} \right\} \quad (7)$$

The calculation results are summarized in the Table 1. They make it possible on reasonable grounds to set the rail track characteristics for the rolling stock modeling according to Lagrange-d'Amber principle. Analysis of results showed that application of reduced mass of track objects in such models is appropriate in cases when there is no complete deflection of the rail occurring at a sufficiently high speed [8], or in cases of simulation of stress-strain state of directly separate elements of the rail track.

Originality and Practical Value

Theoretically proved stiffness and dissipation coefficients of the rail track for calculations of rolling stock dynamics in modern models based on the Lagrange-d'Alembert equation systems are obtained. The established values as opposed to those given in other sources have a substantiated dependence on the track structure and movement speed.

Table 1

Characteristics of railway track as a support when interacting with a wheel

U , MPa	Indicator	Movement speed, km/h					
		80	120	160	200	240	280
21	k_t , MN/m	47.4	48.2	49.5	51.2	53.2	55.3
	β_t , kN s/m	100	70	60	40	40	40
32	k_t , MN/m	62.6	63.1	63.9	65.0	66.2	67.7
	β_t , kN s/m	130	80	70	60	50	50
57	k_t , MN/m	97.2	97.9	98.6	99.4	100.4	101.8
	β_t , kN s/m	210	160	110	90	80	70

Conclusions

The stiffness and dissipation coefficients of the rail track as a support reduced to interaction with the wheel in the models of vehicle movement are determined not only by the elasticity of the under-rail base layers, but also have a direct and inverse dependence on the movement speed, respectively.

The approaches of rail track representation in the rolling stock models, described by the equation systems based on the Lagrange-d'Alembert principle are expanded. The value of rigidity and dissipation coefficients of the railway track depending on its design and speed are obtained.

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ПРЕДСТАВЛЕННЯ ЗАЛІЗНИЧНОЇ КОЛІЇ В МАТЕМАТИЧНІЙ МОДЕЛІ РУХУ ЕКІПАЖІВ

Мета. Задачі моделювання взаємодії колії та рухомого складу є базовими для більшості напрямків сучасних наукових досліджень залізничного транспорту. Складання моделі за принципом Лагранжа-

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д'Аламбера знайшло широке застосування для вирішення задач динаміки рухомого складу. Представлення залізничної колії у моделі руху екіпажів може здійснюватися декількома способами, які, серед іншого, будуть відрізнятися різною деталізацією. Метою даної роботи є створення методики представлення залізничної колії у математичних моделях взаємодії з рухомим складом та отримання практичних результатів для різних характеристик і конструкцій колії на рівні максимальної швидкості. **Методика.** Задача дослідження складається з визначення таких характеристик колії: приведена маса, коефіцієнт жорсткості, коефіцієнт дисипації. Як інструмент для її розв'язання була застосована модель напружено-деформованого стану залізничної колії, основана на поєднанні рівнянь поширення пружної хвилі для опису геометрії обрису частини простору системи, що залучена до взаємодії на даний момент часу, та рівнянь динамічної рівноваги її деформації. Це дає можливість урахування динаміки прогину підрейкової основи, що особливо важливо для умов пасажирського руху, який може відбуватися з високою швидкістю. **Результати.** Отримані теоретично обґрунтовані коефіцієнти жорсткості та дисипації залізничної колії для розрахунків динаміки рухомого складу в сучасних моделях на основі систем рівнянь, складених за принципом Лагранжа-д'Аламбера. Встановлені значення, на відміну від наведених в інших джерелах, мають обґрунтовану залежність, що поєднує конструкції колії та швидкості руху. **Наукова новизна.** Розширені підходи представлення залізничної колії у моделях рухомого складу, описаних системами рівнянь за принципом Лагранжа-д'Аламбера. Розроблена методика визначення характеристик залізничної колії для таких моделей за результатами варіантних розрахунків динамічного прогину рейки від проходження колеса. **Практична значимість.** Авторами отримані значення коефіцієнтів жорсткості й дисипації залізничної колії залежно від її конструкції та швидкості руху для практичного застосування у відповідних моделях взаємодії колії і рухомого складу.

Ключові слова: залізнична колія; взаємодія колії та рухомого складу; модель залізничної колії; жорсткість колії; дисипація колії; динамічний прогин колії; пасажирський рух

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ПРЕДСТАВЛЕНИЕ ЖЕЛЕЗНОДОРОЖНОГО ПУТИ В МАТЕМАТИЧЕСКОЙ МОДЕЛИ ДВИЖЕНИЯ ЭКИПАЖЕЙ

Цель. Задачи моделирования взаимодействия пути и подвижного состава являются базовыми для большинства направлений современных научных исследований железнодорожного транспорта. Составление модели по принципу Лагранжа-д'Аламбера нашло широкое применение для решения задач динамики подвижного состава. Представление железнодорожного пути в модели движения экипажей может осуществляться несколькими способами, которые, среди прочего, будут отличаться детализацией. Целью данной работы является создание методики представления железнодорожного пути в математических моделях взаимодействия с подвижным составом и получение практических результатов для разных характеристик, конструкций пути на уровне максимальной скорости. **Методика.** Задача исследования состоит из определения таких характеристик пути: приведенная масса, коэффициент жесткости и коэффициент диссипации. В качестве инструмента для ее решения была использована модель напряженно-деформированного состояния железнодорожного пути, основанная на совместном использовании уравнений распространения упругих волн для описания геометрии очертания части пространства системы, которая задействована во взаимодействии на данный момент времени, и уравнений динамического равновесия ее деформации. Это дает возможность учесть динамику прогиба подрельсового основания, что особенно важно для условий пассажирского движения, которое может осуществляться с высокой скоростью. **Результаты.** Получены теоретически обоснованные коэффициенты жесткости и диссипации железнодорожного пути для расчетов динамики подвижного состава в современных моделях на основе систем уравнений, составленных по принципу Лагранжа-д'Аламбера. Установленные значения, в отличие от приведенных в других источниках, имеют обоснованную зависимость, которая объединяет конструкции пути и скорости движения. **Научная новизна.** Расшире-

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ны подходы представления железнодорожного пути в моделях подвижного состава, описанных системами уравнений по принципу Лагранжа-д'Аламбера. Разработана методика определения характеристик железнодорожного пути для таких моделей по результатам вариантных расчетов динамического прогиба рельса от прохождения колеса. **Практическая значимость.** Авторами получены значения коэффициентов жесткости и диссипации железнодорожного пути в зависимости от конструкции и скорости движения для практического использования в соответствующих моделях взаимодействия пути и подвижного состава.

Ключевые слова: железнодорожный путь; взаимодействия пути и подвижного состава; модель железнодорожного пути; жесткость пути; диссипация пути; динамический прогиб пути; пассажирское движение

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